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No. 655

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PRINCIPLES INVOLVED IN THE COOLING OF A  
FINNED AND BAFFLED CYLINDER

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SUMMARY

An analysis of the cooling problem for a finned cylinder is made on the basis of the known fundamental principles of heat transfer from pipes. Experimental results that support the analysis are presented. The results of previous investigations on the problem are evaluated on the basis of the analysis and the results. An illustration of the application of these principles to a specific problem is included.

INTRODUCTION

The N.A.C.A. has made a study of the effect of fin spacing on model baffled engine cylinders (references 1, 2, and 3). The purpose of this report is to present further information relating to the subject.

An analysis of cooling phenomena, using the known principles of heat transfer for pipes, is made and the analysis is used to correlate the results of the present study with the results presented in references 1 and 2.

ANALYSIS OF THE PROBLEM

In order to understand more clearly the fundamental principles involved in the cooling of a finned cylinder, it is useful to consider the simpler case of the heat transfer to a fluid flowing through a pipe. McAdams (reference 4) gives the nondimensional formula



$$\frac{hd}{k} = C_1 \left( \frac{c_p \mu}{k} \right)^m \left( \frac{Vd\rho}{\mu} \right)^n \left( \frac{l}{d} \right)^i \quad n < 1 \quad (1)$$

where  $h$  is the local heat-transfer coefficient.

$d$ , the hydraulic diameter of the tube.

$k$ , the conductivity of the fluid.

$c_p$ , the specific heat of the fluid.

$\mu$ , the viscosity of the fluid.

$V$ , the average velocity of the fluid.

$\rho$ , the density of the fluid.

$l$ , the length of the pipe.

$C_1$ , a constant.

The exponents  $m$ ,  $n$ , and  $i$  have two sets of values, one for turbulent and one for laminar flow. The exponent  $i$  is very small and thus the ratio  $l/d$  has only a small influence. Prandtl's number  $c_p \mu / k$  is a measure of the physical properties of the fluid and is almost constant for a given fluid.

The relation for  $h$  may be written

$$h = C_2 \frac{V^n}{d^{1-n}} \quad n < 1 \quad (2)$$

This relation indicates that the heat-transfer coefficient will increase with the velocity and increase with a decrease in the pipe diameter. If the wall of the pipe is maintained at a constant temperature, the fluid is heated up and, if the pipe is sufficiently long, the average fluid temperature approaches the wall temperature, as illustrated in figure 1.

If the pipe is required to dissipate a constant amount of heat per unit length and there is no conduction along the pipe, the temperature of tube and fluid varies with pipe length,  $l$ , as illustrated in figure 2.

The heat-transfer coefficient is proportional to  $v^n/d^{1-n}$ . Thus it is constant throughout the pipe and, as the fluid heats up, the pipe must heat up also. This illustration pictures almost exactly the conditions imposed when cooling is accomplished by a baffled finned cylinder. The air passages formed by the fins, the cylinder wall, and the baffle are in reality only pipes through which the average velocity must remain constant. If the velocity distribution is uniform,  $h$  will be maximum for that average velocity. It is known from reference 5 that, throughout the greater part of the length, the velocity distribution does not change to any appreciable extent so that  $h$  must remain relatively constant except near the entrance to and exit from the air passages. The fact that the fins transmit the heat by conduction from the cylinder wall to the air and thus represent indirect cooling does not enter the picture except as an efficiency factor. With a given arrangement, the fin temperature and the air temperature follow curves similar to those in figure 2 along the air-passage length.

#### Effect of Velocity and Tube Diameter

It has been shown that the heat-transfer coefficient can be increased by increasing the velocity of the fluid or by decreasing the diameter of the tube. Since the exponent  $n$  is approximately 0.8 for turbulent flow, it is obvious that the heat-transfer coefficient will be affected much more by changes in velocity of the fluid than by changes in the diameter of the pipe. This observation would seem to make the solution of the problem very simple. Blower cooling is just such a solution.

If the diameter of the tube is decreased, the heat-transfer coefficient is increased proportionally to  $1/d^{1-n}$  giving  $1/d^{0.2}$  for turbulent flow. This increase in itself is not particularly spectacular. On a finned cylinder when  $d$ , the hydraulic diameter, is decreased, however, the number of fins is increased in inverse proportion. Thus the area to which  $h$  is applied is greatly increased and the heat that may be dissipated is almost doubled by halving  $d$ .

#### Power Required for Cooling

The power required for cooling is  $Q\Delta p$ , where  $Q$  is

the volume of fluid (per unit time) and  $\Delta p$  the pressure drop across the pipe. The volume and pressure drop are functions of the velocity. Glauert (reference 6) gives the relation for turbulent flow:

$$\frac{dp}{dl} = C_1 \frac{1}{R^{1/4}} \frac{\frac{1}{2} \rho V^2}{d} \quad (3)$$

where  $dp/dl$  is the pressure gradient in the pipe and  $R$  is the Reynolds Number  $Vd\rho/\mu$ . The pressure gradient is thus a function of the pipe diameter and the fluid velocity. For a given pipe,  $\Delta p \propto V^{1.75}$ , where  $\Delta p$  is the actual pressure drop in the pipe. The  $\Delta p$  loss at the entrance and the exit is the same as for an orifice and will therefore be proportional to  $V^2$ . Thus doubling the velocity will increase  $h$  75 percent (see equation (2)) and require about seven times the power for cooling (equation (3)). The foregoing relationship among  $\Delta p$ ,  $l$ ,  $d$ , and  $V$  is suited to practical work because  $\Delta p$  is given explicitly in terms of the other variables. The  $\Delta p$  available for cooling being fixed, the other variables must be chosen to give adequate cooling and to be consistent among themselves.

#### Heat Dissipation

The heat dissipated  $\Delta H$  in terms of the pipe dimensions and pressure drop can now be written

$$\Delta H = Q \rho c_p d T_A = h \pi d dl (T_w - T_a) \quad (4)$$

where  $T_w$  is the pipe-wall temperature.

$T_a$  is the air temperature at any point  $l$ .

$$Q = \frac{V \pi d^2}{4}$$

The inlet-air temperature is designated  $T_{ia}$ .

$$\int_{T_{ia}}^{T_a} \frac{dT_a}{T_w - T_a} = \int_0^l \frac{4h \, dl}{Vd\rho c_p}$$

$$\log \frac{T_w - T_a}{T_w - T_{ia}} = - \frac{4hl}{Vd\rho c_p}$$

$$\frac{T_w - T_a}{T_w - T_{ia}} = e^{-\frac{4hl}{Vd\rho c_p}}$$

or

$$\frac{T_a - T_{ia}}{T_w - T_{ia}} = 1 - e^{-\frac{4hl}{Vd\rho c_p}} = \eta_T \quad (5)$$

As the heat-transfer efficiency  $\eta_T$  approaches zero, the heat transfer goes to a maximum.

The basic problem is the determination of the heat that can be dissipated with a given frontal area, base area, pressure drop, and temperature difference between the cooling air and the body. The use of equation (5) in the second part of equation (4) gives  $H$  as a function of the tube dimensions, the velocity, and the difference between inlet-air and tube-wall temperatures.

$$H = \frac{V\pi d^2}{4} \rho c_p (T_w - T_{ia}) \left(1 - e^{-\frac{4h}{Vd\rho c_p}}\right)$$

Eliminating  $h$  and  $V$  by means of equations (2) and (3), respectively,

$$H = \frac{0.704 \Delta p^{4/7} d^{19/7} \rho^{4/7}}{l^{4/7} \mu^{1/7}} c_p (T_w - T_{ia})$$

$$\left( 1 - e^{-\frac{C_3 \mu^{1/35} \rho^{8/35} l^{55/35}}{\Delta p^{4/35} d^{53/35}}} \right) \quad (6)$$

Thus  $H$  is a function of the temperature difference, the  $\Delta p$ , and the pipe dimensions. The curve of  $H/A$  against  $l$  (fig. 3) for constant  $\Delta p$  and constant temperature difference between inlet air and pipe wall has a maximum at a given  $l/d$  ratio. These curves show that the heat dissipated from a given frontal area is almost the same, regardless of tube diameter as long as the length is made optimum for each diameter.

The same analysis holds true for the air passage on a finned cylinder, where  $d$  is the hydraulic diameter. The actual constants appearing in the equations will be different but the fundamental fact that there is an appropriate length of air passage for each fin spacing is unaffected.

The actual values of the heat-transfer coefficient are of more academic interest than practical value. The real problem is to know what changes to make in finning and baffling to give a desired increase in cooling.

The analysis has been based largely on the case of turbulent boundary layer. The case for laminar boundary layer is essentially the same, in which

$$h \propto \frac{V^{0.5}}{d^{0.5}} \quad dp \propto \frac{lV}{d^2}$$

At low velocity, the flow through a given pipe will be laminar and at high velocity the flow will be turbulent. It follows, then, that there must be a transition between

these two flow types that will show up in both the friction losses and the heat-transfer coefficient.

## RESULTS

### Tests on Pipes

A series of tests was run with pipes in connection with another problem to determine the friction loss. These tests were made with tubes ranging from 1/16-inch diameter and 24-inch length to 1/2-inch diameter and 72-inch length. Similar tests were also made of tubes having square and rectangular cross sections. Figure 4 shows the friction

factor  $\left(f = \frac{dp}{dl} \frac{d}{q}\right)$  plotted against the Reynolds Number.

The results are in complete agreement with those of Nikuradse (reference 7) for smooth tubes.

The results for tubes fall on two straight lines connected by transition curves. The straight line at the left is for laminar flow and the straight line at the right is for turbulent flow. The various transitions between these two types of flow depend upon the entrance, the roughness, and the straightness of the tube. Transition must come between values of Reynolds Numbers A and B shown in figure 4. It may occur at A if the entrance is bad or there are sharp bends in the tube or if the inside of the tube is rough. It cannot come later than B regardless of perfect entrance, straight pipe, and smooth inside pipe surface. (See reference 8.) Any practical arrangement, such as a radiator or finned cylinder, gives a transition similar to C. A transition that occurs at a Reynolds Number close to B is extremely unstable and can be realized experimentally only with very elaborate tests.

Figure 5 shows a curve of  $h$  against  $V$  at constant pipe diameter and figure 6 shows  $h$  against  $d$  at constant velocity. Each figure is for a transition like C (fig. 4). A study of figures 4 and 6 reveals that, for constant fluid velocity, both friction and heat transfer increase as the diameter of the pipe decreases. Thus, as the pipe diameter is decreased, the friction increases and the velocity would be reduced unless the pipe length were reduced. As the pipe diameter is reduced, both the area for



cooling per unit length and the heat-transfer coefficient increase so that as much heat can be dissipated from the short pipe as from the long pipe when the diameter is made optimum for the length. It is seen that, whereas the  $l/d$  ratio has little effect on the local heat-transfer coefficient, it is all important in determining the total heat that may be dissipated from a given surface. A similar analysis led to the ideas proposed in reference 9.

### Tests on Finned Cylinders

A series of tests was run on sections of finned cylinders in which the length-spacing ratios were constant. Table I shows the dimensions of the elements used.

TABLE I

Length arc (deg.)	s fin spacing (in.)	t fin thickness (in.)	Number of fins tested
180	0.125	0.033	6
90	.0625	.025	12
45	.031	.016	21
22.5	.016	.006	45

All fins were of brass 1 inch in width. The cylinder diameter, measured to the base of the fins, was 5.81 inches. The heat-transfer coefficient  $U$  was determined for the base area  $A$ . The coefficient  $U$  is shown plotted against velocity (fig. 7)

$$U = \frac{H}{A \Delta T(w - a) l_m}$$

The expression  $\Delta T(w - a) l_m$  is the logarithmic mean of the

difference between the base temperature and the cooling-air temperature.

Reference 10 gives the relation

$$U = \frac{h}{s + t} \left[ \frac{2}{a} \left( 1 + \frac{w}{2R} \right) \tanh h a w + s \right]$$

where  $a$  is  $\sqrt{2h/k_m t}$ ,  $k_m$  is the thermal conductivity of the fins, and  $w$  is the fin width. From this relation for  $U$  it is possible to find the average value of  $h$ . Obviously  $h$  determined in this way will be of rather low accuracy since all the errors of measurement are cumulative. Figure 8 shows Nusselt's number  $hd/k$  plotted against Reynolds Number,  $Vd\rho/\mu$ . The scatter of the data is considerable but there is no doubt about the data having the usual ability to be correlated on such a plot. Figures 7 and 8 both show results that are identical in every respect with those published for pipes. These figures show results covering both the laminar and turbulent ranges together with the transition from laminar to turbulent flow.

Figure 9 was constructed to show how the several variables affect the over-all heat-transfer coefficient  $U$ . Here  $U$  is plotted as a function of  $s$ , each curve being for a particular fin width. The local heat-transfer coefficients were taken from figure 8 at a velocity of 100 miles per hour. The optimum fin thickness was used for each spacing and width. Figure 10 shows the optimum fin thickness plotted against fin spacing for the four fin widths. A similar set of curves can be made for any air speed.

#### DISCUSSION

These results demonstrate that the fundamental principles of heat transfer as found for pipes may be applied with confidence to problems of heat transfer on finned cylinders. The problem of application comes in the design of air entrance and exit to the fin space. (See reference 9.)

When the results presented in references 1, 2, and 3, are viewed in the light of the preceding analysis, it is

apparent that the optimum spacing resulted from the choice of fixed air-passage length. Obviously, smaller spacings would have been found for shorter air-passage lengths.

The heat-transfer coefficients defined in these tests had an inverse dependence upon the heating up of the air; that is, as the air heated up, the coefficient decreased. Although such a coefficient may be used to describe the results of a given test, it is impossible to use the coefficient for purposes of prediction of results when any of the test conditions are changed. Accordingly, great care should be exercised in the use of these data to make certain that the arrangement to which they are to be applied is identical in all respects to the one tested.

#### Application of the Principles

The solution of a given cooling problem can be solved most easily by comparison with some arrangement whose cooling properties are known. If information on the increase in cooling needed on one arrangement is known, it is necessary only to change the fin dimensions to give the desired increase. No serious error will result from assuming  $h$  to be constant, since it varies with  $d^{-0.2}$  for turbulent flow and  $d^{-0.5}$  for laminar flow. Present fin spacings and air speeds are invariably in the turbulent-flow range. The increase in pressure drop as spacing is decreased can be estimated by using the relations from reference 6.

In order to illustrate how a cooling problem may be solved by comparison, assume a steel engine cylinder having fins 1/2 inch wide with 0.100-inch spacing and 0.0375-inch thickness with a baffle in contact with the rear half of the cylinder. Suppose an increase in cooling of 25 percent is desired for the baffled part of this cylinder. The curves of figures 9 and 10 are for steel fins and are therefore directly applicable. The original cylinder had a heat-transfer coefficient  $U = 1.6$ ; therefore, a 25-percent increase in cooling requires a  $U = 2.0$ .

If the increase in  $U$  is obtained by increasing the fin width, it is necessary to make a cross plot of figures 9 and 10 to give  $U$  against  $w$  on figure 11(a) and  $t$  against  $w$  on figure 11(b), both at  $s = 0.10$ . From figure 11(a), a  $U = 2.0$  occurs at  $w = 0.9$  inch and from figure 11(b) a  $w$  of 0.9 requires a thickness of 0.061

inch. The velocity between the fins will remain the same as in the original arrangement but there will be a 55-percent increase in open area, resulting in a 55-percent increase in air for cooling.

If the spacing is reduced, keeping the width constant and reducing the baffle length proportional to the spacing, another solution is obtained. In this case, using figure 9 and following along the line for 1/2-inch fin width to the point where  $U$  is 2.0, a spacing of 0.075 inch is found. Figure 10 gives a fin thickness of 0.033 inch for this spacing and width. With this arrangement, the mass flow of air and the heat-transfer efficiency both remain practically unchanged.

The two solutions obtained by changing width and spacing illustrate the complete interdependence of all the variables. The solution by changing width gave an increase of 25 percent in cooling for the original base area at an increase in power to cool of 55 percent. The solution by changing spacing gave a 25-percent increase in cooling in 75 percent of the base area at the same power to cool. Both solutions require minor changes in the baffle. These two solutions are only illustrative of an infinite number of possible solutions. They do, however, illustrate the manner in which the principles of heat transfer as known for pipes may be applied to the problem of cooling a finned cylinder.

Langley Memorial Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., May 11, 1938.

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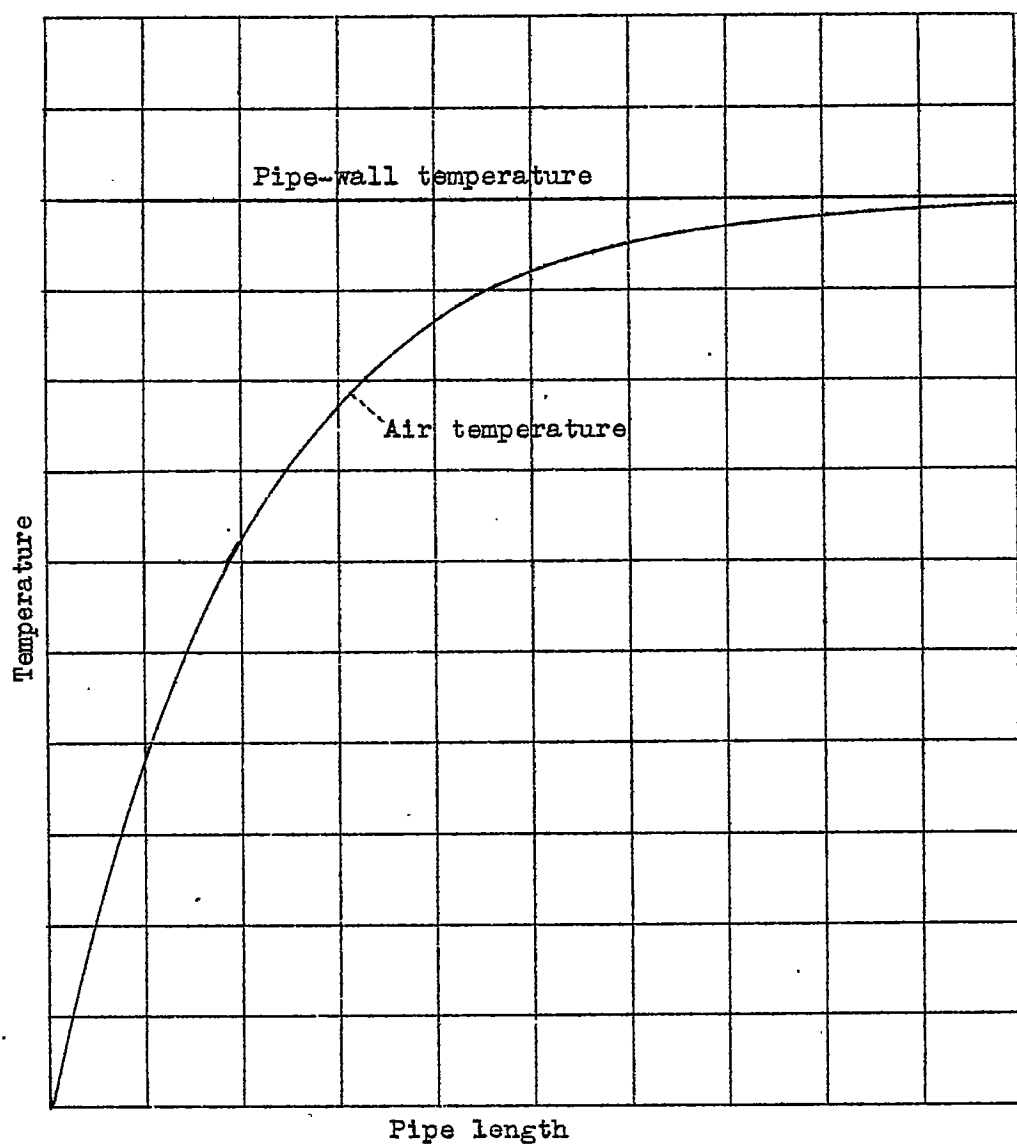


Figure 1.- Air temperature rise along pipe with constant wall temperature.

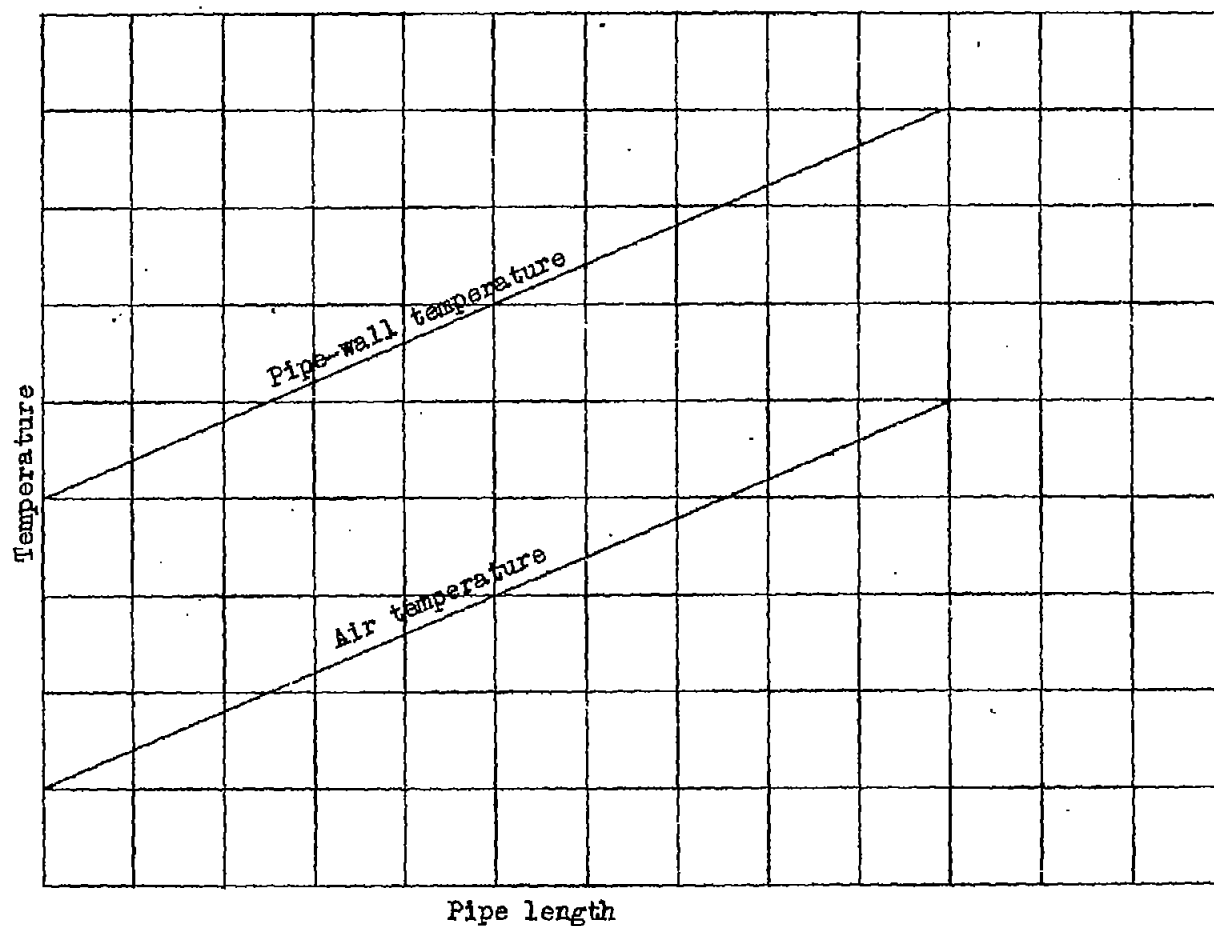


Figure 2.- Variation of air and wall temperatures with pipe length for constant heat input per unit pipe length.

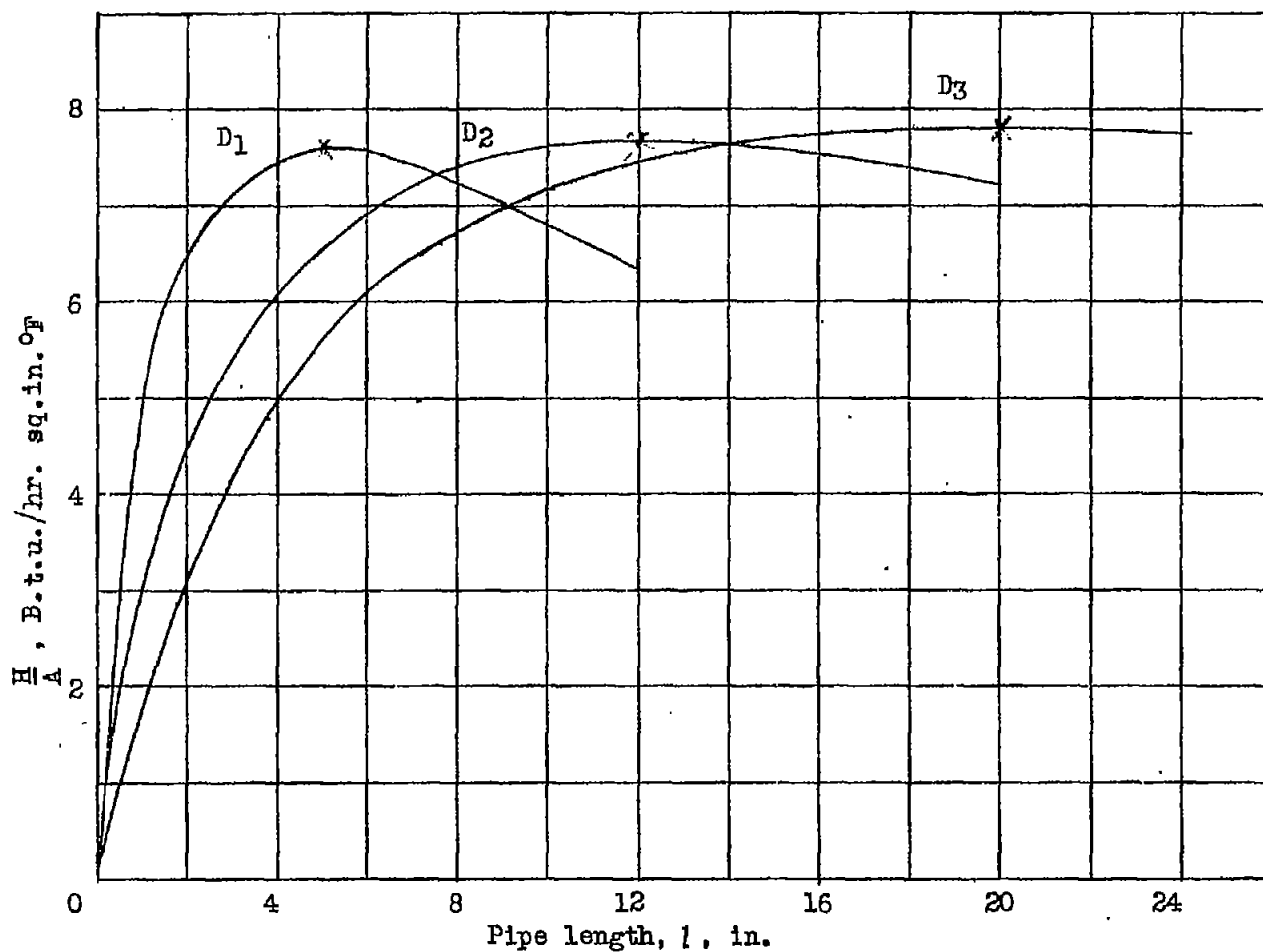


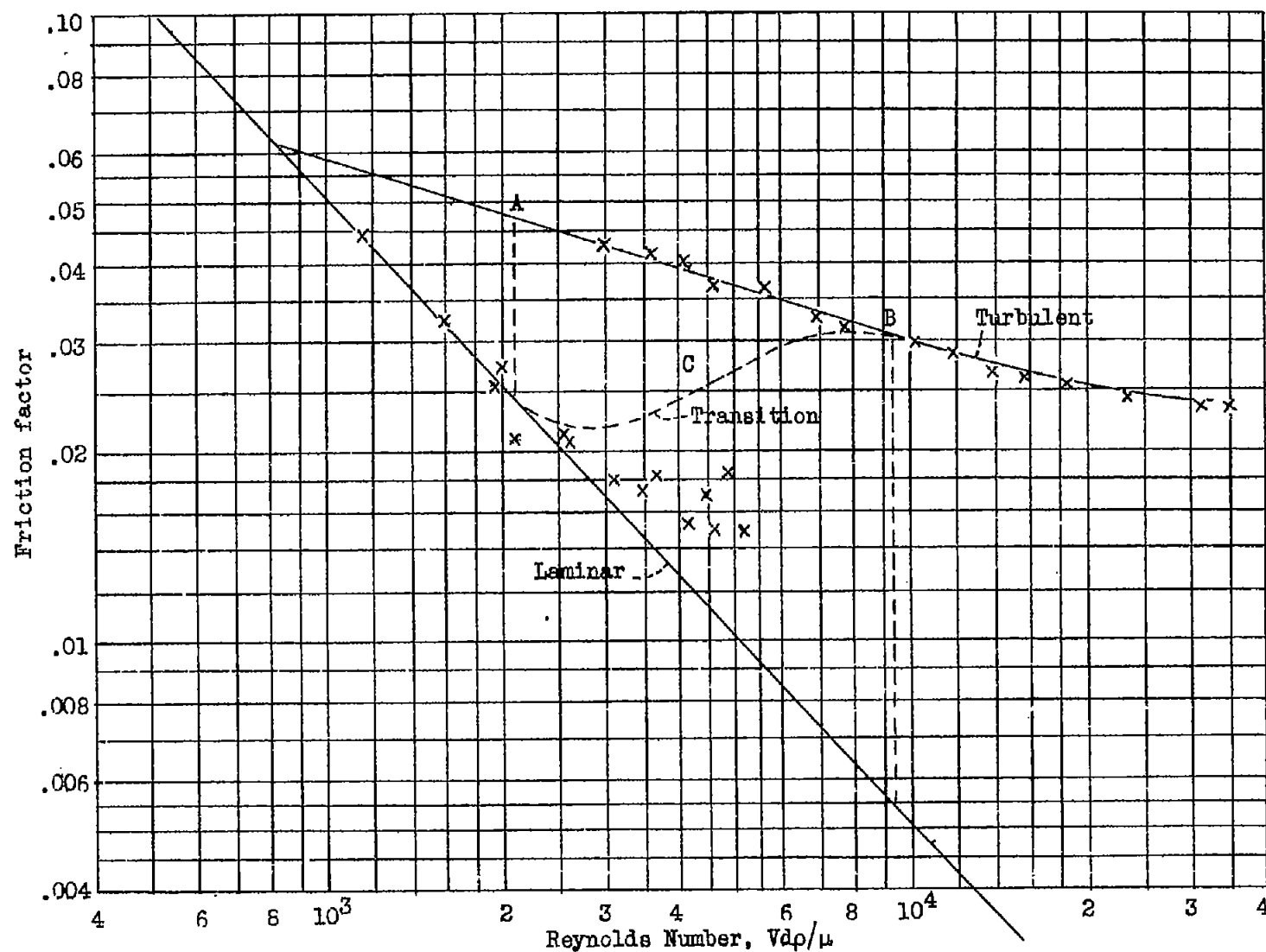
Figure 3.- Variation of heat dissipated per unit frontal area with pipe length for three pipe diameters,  $D_1 < D_2 < D_3$

$D_1 =$

$D_2 =$

$D_3 =$





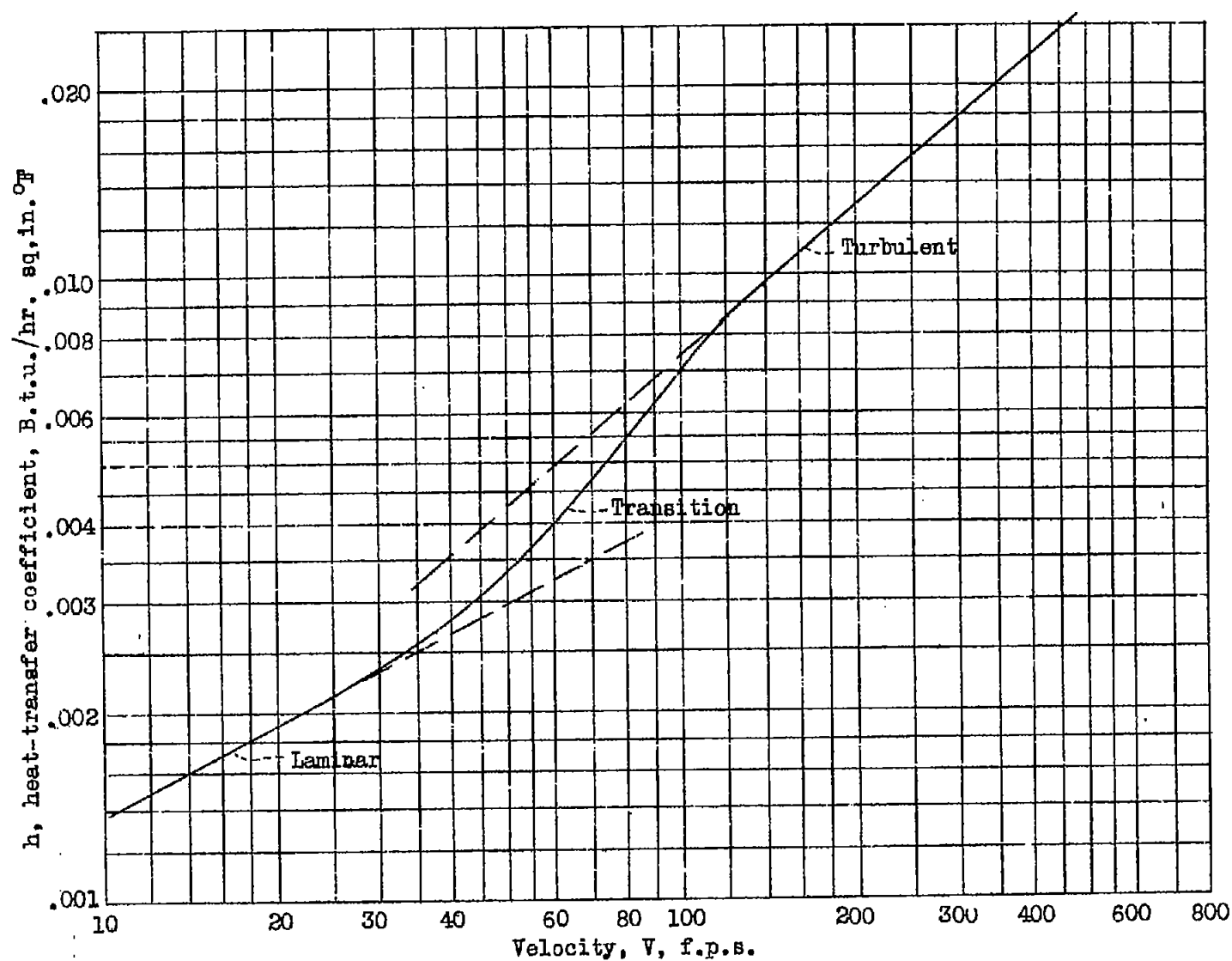


Figure 5.- Variation of heat-transfer coefficient with velocity at constant diameter.

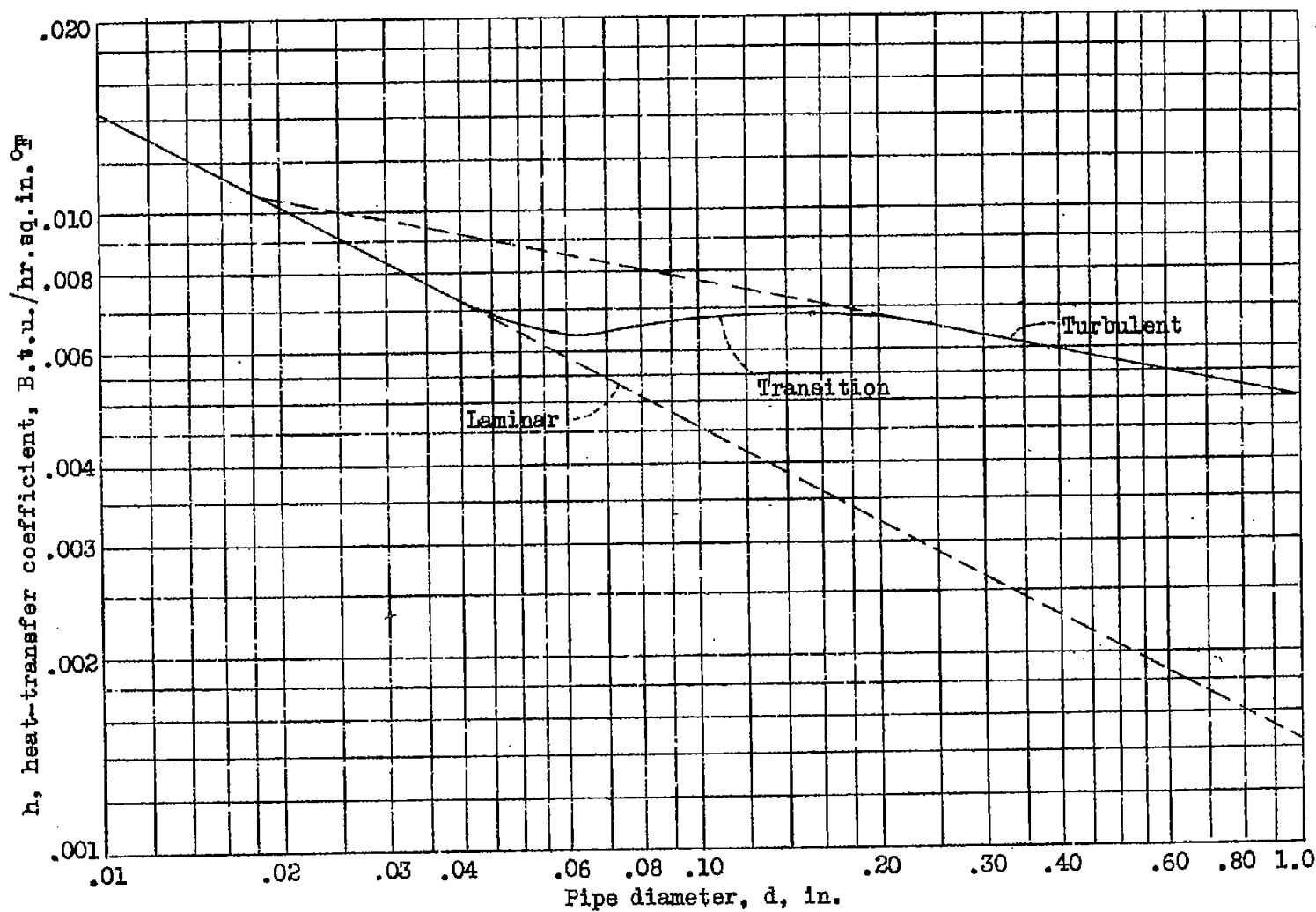


Figure 6.- Variation of heat-transfer coefficient with pipe diameter at constant velocity.

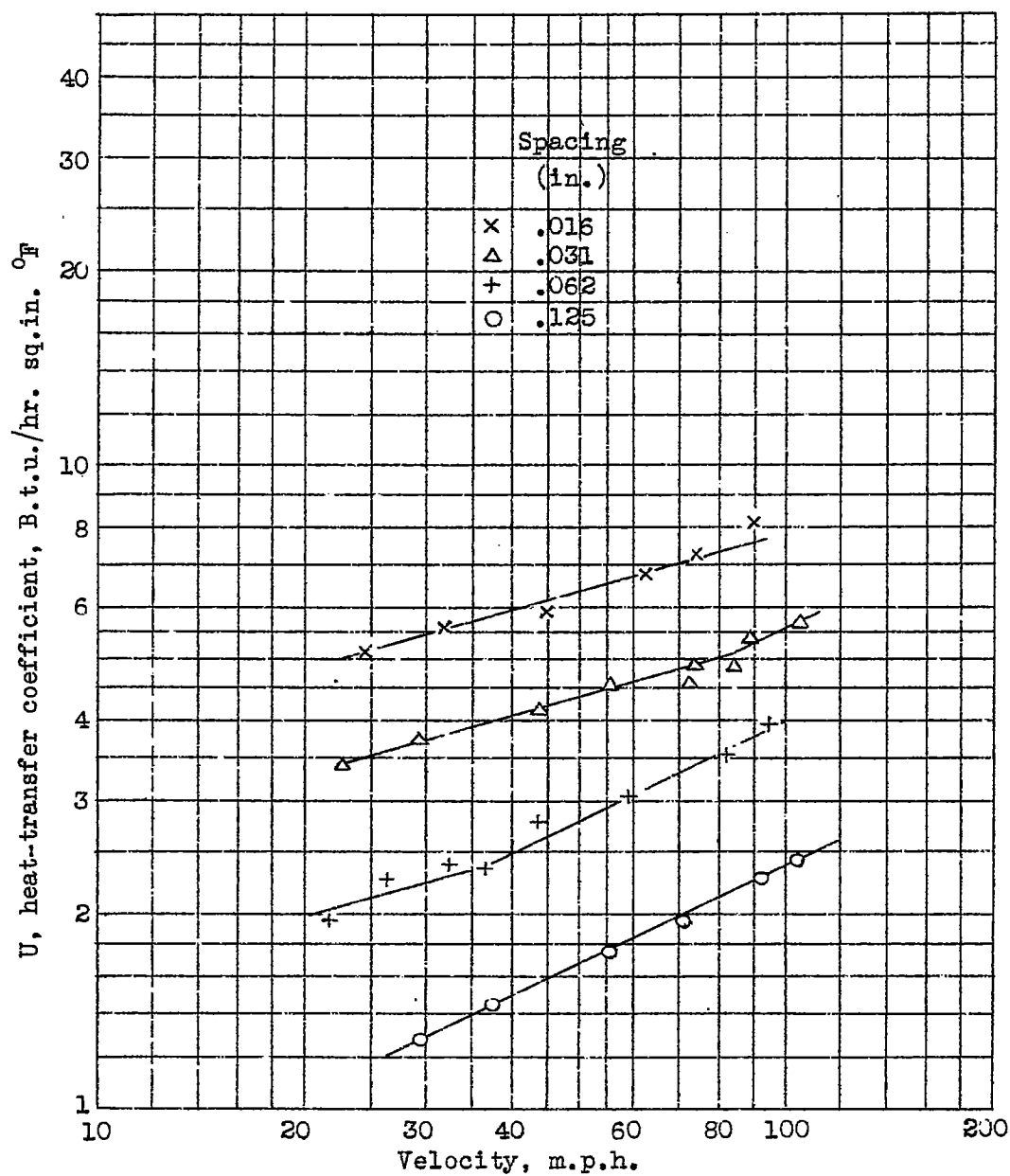


Figure 7.- Variation of heat-transfer coefficient with velocity for several fin spacings.

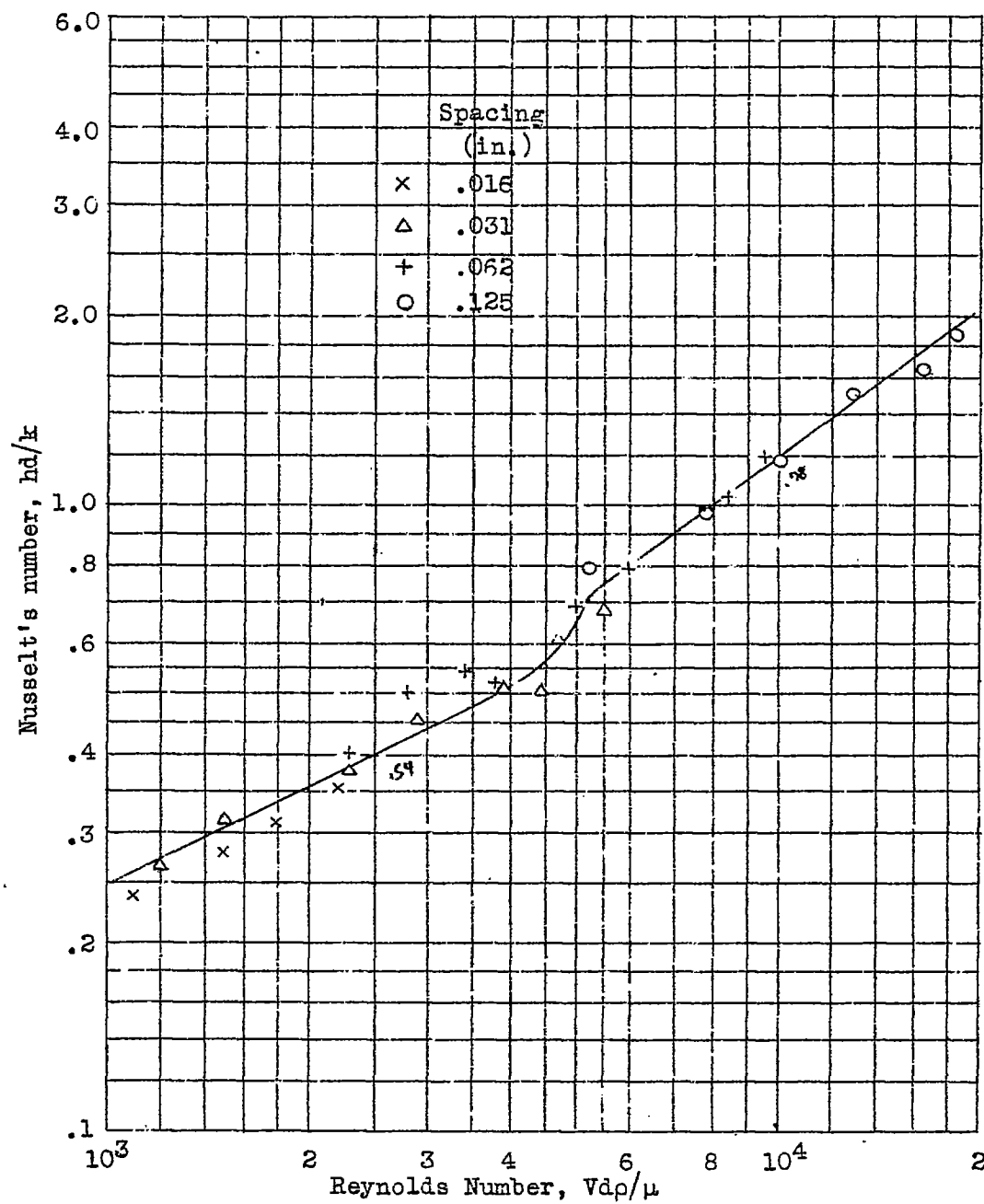


Figure 8.- Variation of Nusselt's number with Reynolds Number for several spacings.

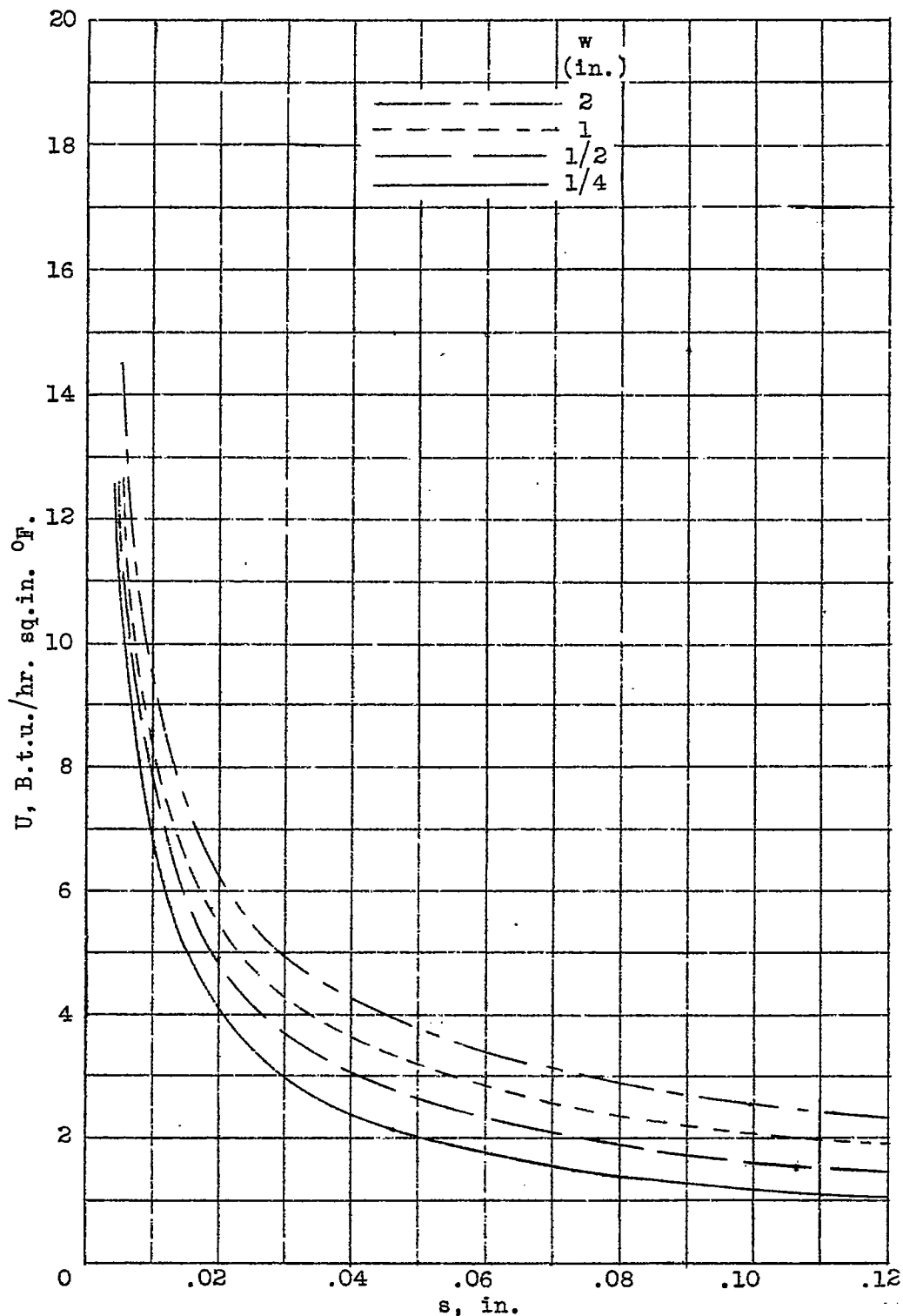


Figure 9.- Variation of over-all heat-transfer coefficient with spacing for four fin widths, at 100 m.p.h. and optimum fin thickness. Steel cylinder;  $k_m = 2.17$  B.t.u./hr.sq.in.°F.

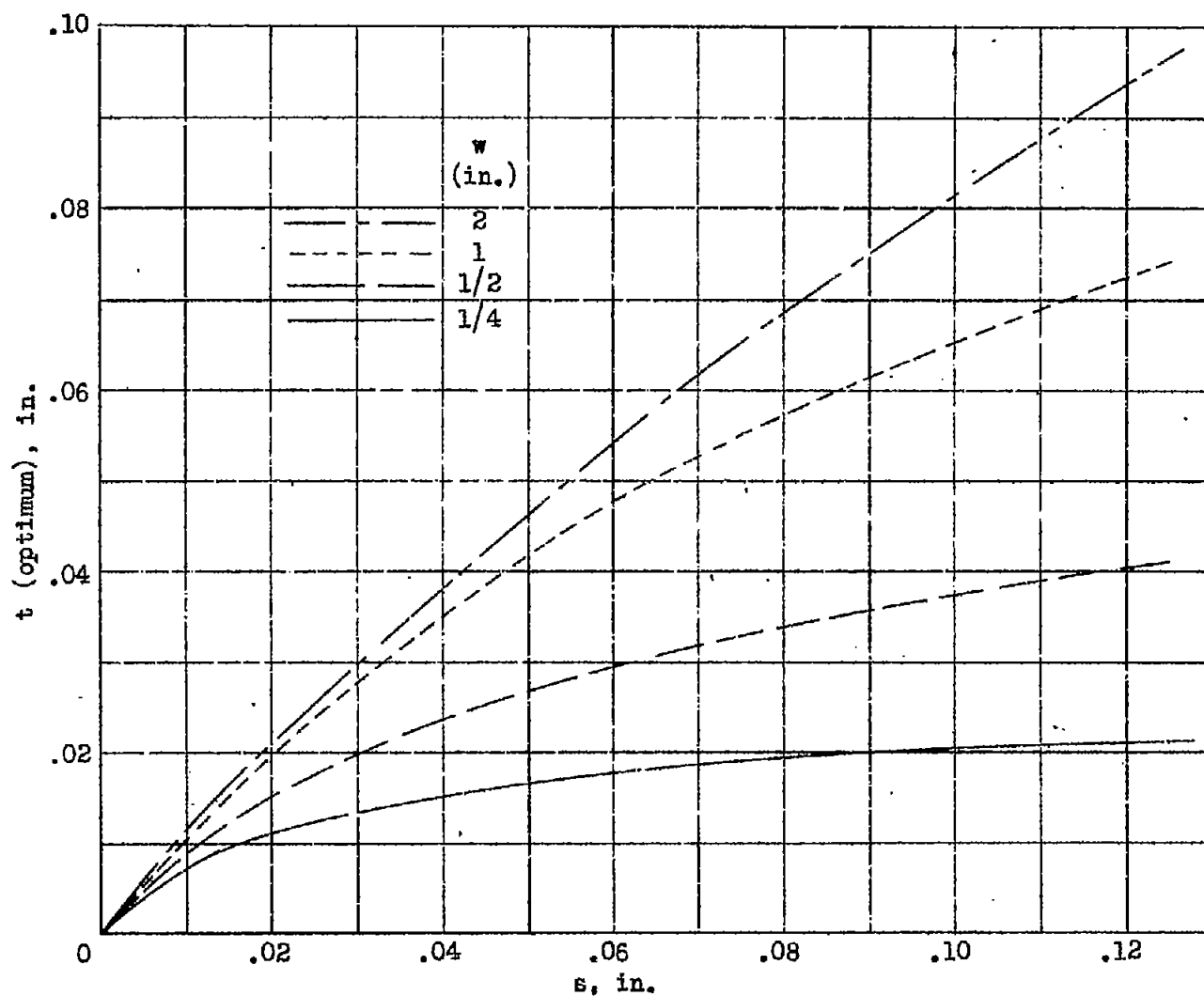


Figure 10.- Variation of optimum fin thickness with spacing for four fin widths.  
Steel cylinder;  $k_m = 2.17$  B.t.u./hr.sq.in. $^{\circ}\text{F}$

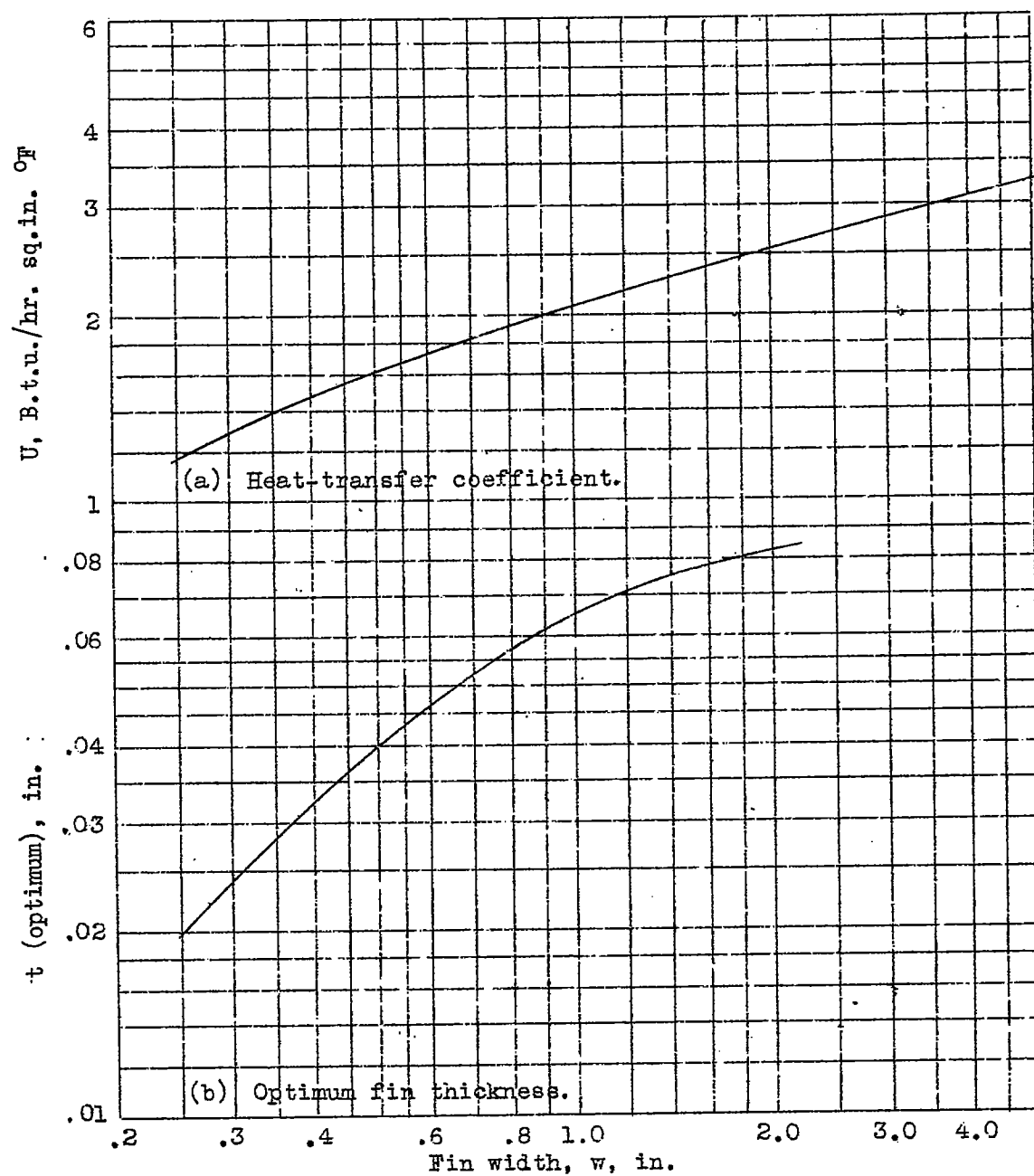


Figure 11.- Variation of cylinder characteristics with fin width at 0.10 in. fin spacing.